Inverse of Natural Log *(ln and e)*

Log Form: $y = \ln x$

What is the inverse of $y = \ln x$?

Switch x and y $\quad x = \ln y$

Write as exponent *"Exponentiate"* $\quad e^x = e^{\ln y}$

Solve for y $\quad e^x = y$

$y = e^x$

Remember: ln could also be written as $\log_e$

Ex1: Find the inverse of $y = e^{x-2}$

$x = e^{y-2}$

~~log~~ $\ln x = \ln e^{y-2}$

$\ln x = y - 2$

$+2 \quad +2$

$\ln(x) + 2 = y$

Ex2: Find the inverse of $y = \ln(x - 4) + 2$

$x = \ln(y-4) + 2$

$x - 2 = \ln(y-4)$

$e^{x-2} = e^{\ln(y-4)}$

$e^{x-2} = y - 4$

$e^{x-2} + 4 = y$

Graph Exponential & Log

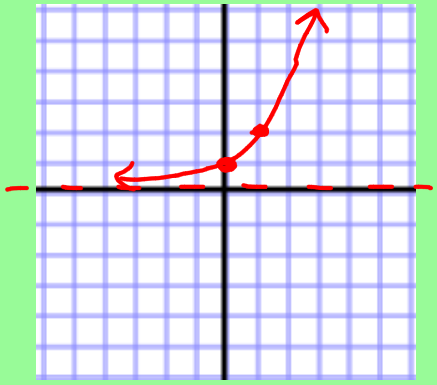
Exponential: $y = b^x$

(0 , 1)

(1 , b)

Always on the graph.

$y = 2^x$ (0,1) (1,2)



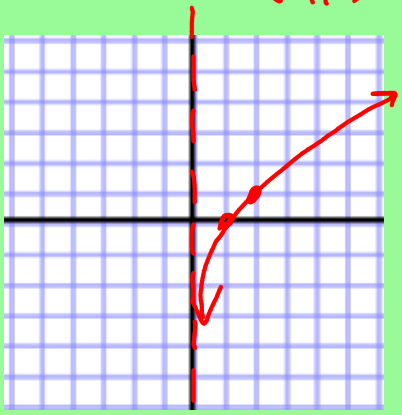
Log: $y = \log_b x$

(1 , 0)

(b , 1)

Always on the graph.

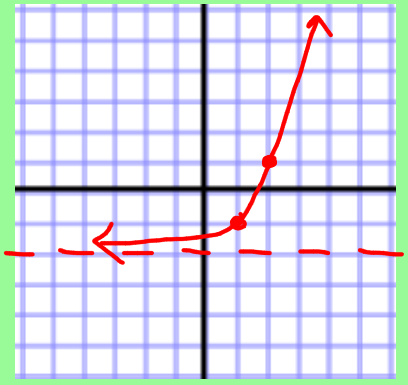
$y = \log_2 x$ (1,0) (2,1)



horizontal asymptote at k

Exponential $y = b^{(x-h)} + k$

$y = 3^{(x-1)} - 2$ ← 1 right, 2 down



(0 , 1) ⇒ (1, -1)

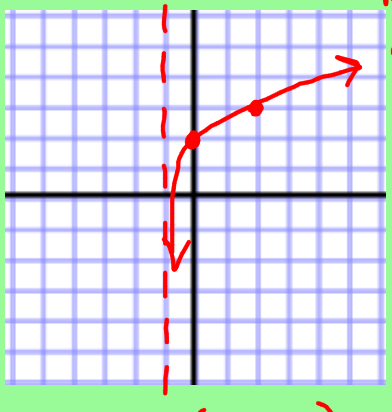
(1 , ?) ⇒ (2, 1)

Always on the graph.

vertical asymptote at h

Log $y = \log_b (x - h) + k$

$y = \log_3 (x + 1) + 2$ ← 1 left, 2 up



(1 , 0) ⇒ (0, 2)

(? , 1) ⇒ (2, 3)

Always on the graph.

27) $y = \log_4 3^x$

$x = \log_4 3^y$

$4^x = 4^{\log_4 3^y}$

$4^x = 3^y$

$\log_3 4^x = \log_3 3^y$

$\log_3 4^x = y$

5) $y = \log_x 3$

$x = \log_y 3$

$y^x = y^{\log_y 3}$

$\sqrt[x]{y^x} = \sqrt[x]{3}$

$y = \sqrt[x]{3}$ OR $3^{\frac{1}{x}}$

Additional examples from the homework worksheet.

14) $y = \log_{\frac{1}{3}}(x-5)$

$x = \log_{\frac{1}{3}}(y-5)$

$\left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{\log_{\frac{1}{3}}(y-5)}$

$\left(\frac{1}{3}\right)^x = y-5$

$\left(\frac{1}{3}\right)^x + 5 = y$

33) $y = \dfrac{7 \cdot 3^x + 1}{3^x}$

$x = \dfrac{7 \cdot 3^y + 1}{3^y}$

$x = \dfrac{7 \cdot 3^y}{3^y} + \dfrac{1}{3^y}$

$x = 7 + \dfrac{1}{3^y}$

$x - 7 = \left(\frac{1}{3}\right)^y$

$\log_{\frac{1}{3}}(x-7) = \log_{\frac{1}{3}}\left(\frac{1}{3}\right)^y$

$\log_{\frac{1}{3}}(x-7) = y$